MOMENTUM PRINCIPLE

The Momentum Equation for Cartesian Coordinates

X-direction:

$$\sum F_X = \frac{d}{dt}\int_{cv} v_X \rho dQ + \sum_{CS}(\dot{m}v)_{outX} - \sum_{CS}(\dot{m}v)_{inX}$$

Y-direction:

$$\sum F_Y = \frac{d}{dt}\int_{cv} v_Y \rho dQ + \sum_{CS}(\dot{m}v)_{outY} - \sum_{CS}(\dot{m}v)_{inY}$$

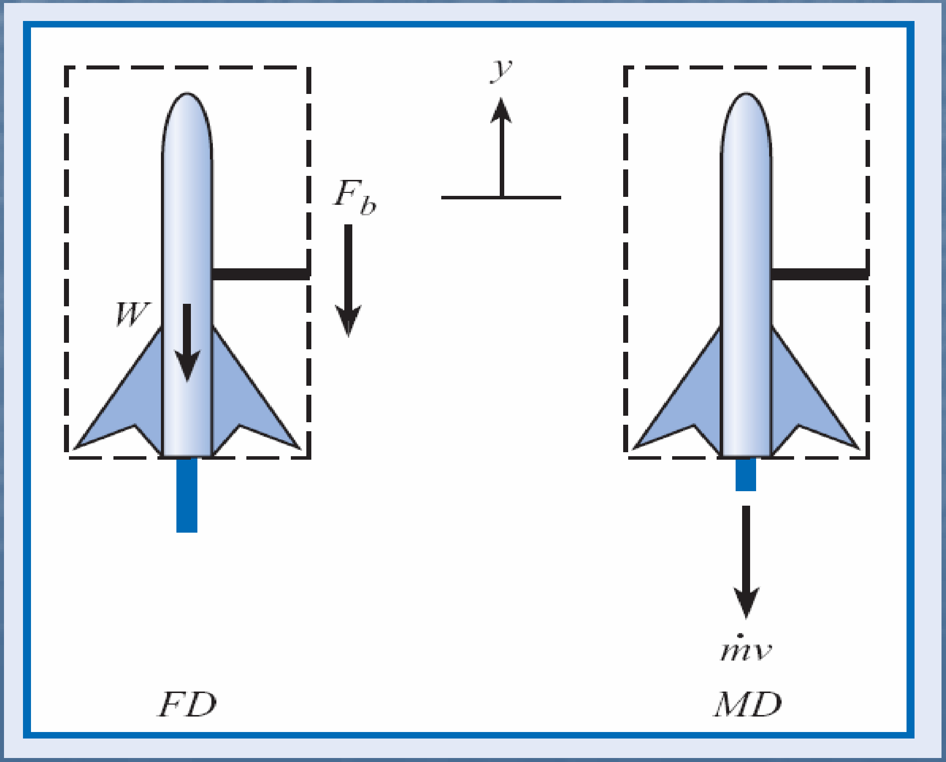
Z-direction:

$$\sum F_Z = \frac{d}{dt}\int_{cv} v_Z \rho dQ + \sum_{CS}(\dot{m}v)_{outZ} - \sum_{CS}(\dot{m}v)_{inZ}$$

Typical Application of Momentum Equation

Fluid Jets **Example 6.1**

The sketch below shows a 40-g rocket, of the type used for model rocketry, being fired on a test stand in order to evaluate thrust. The exhaust jet from the rocket motor has a diameter of $d = 1$ cm, a speed of $v = 450$ m/s, and a density of $\rho = 0.5\ \mathrm{kg/m^3}$. Assume the pressure in the exhaust jet equals ambient pressure, and neglect any momentum changes inside the rocket motor. Find the force F_b acting on the beam that supports the rocket.



Find the force (F_b) acting on the beam that supports the rocket?

To solve the problem, always consider two diagrams:

1. **Force diagram (FD)**
2. **Momentum diagram (MD)**

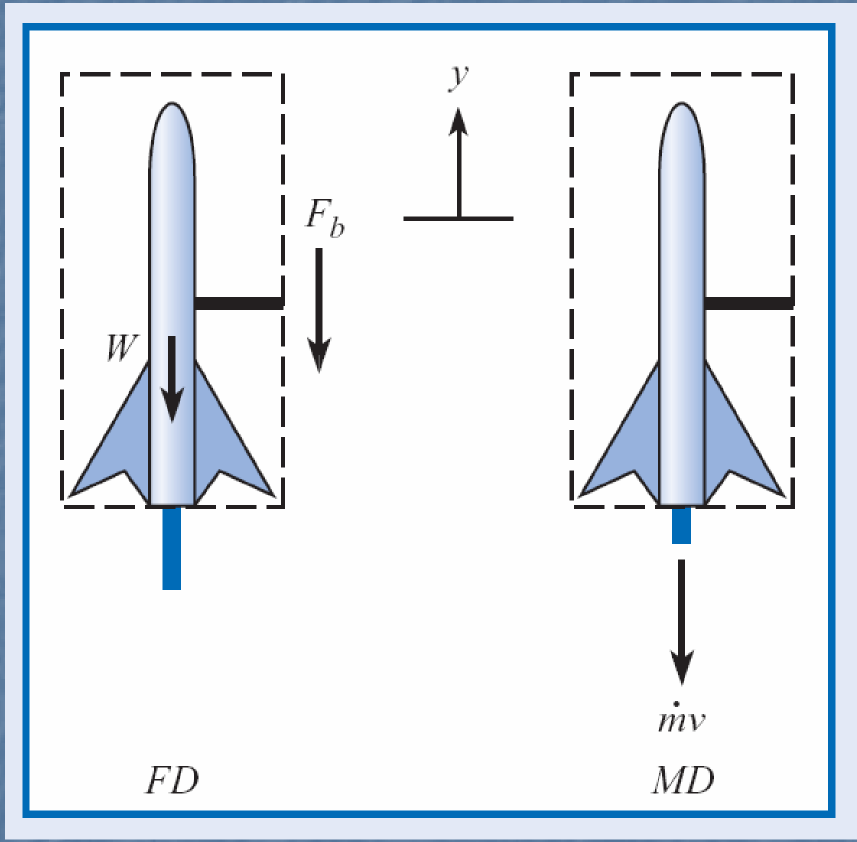
Consider positive direction upwards)

From the force diagram in the Z-direction

$$\sum F_z = -(W + F_b)$$

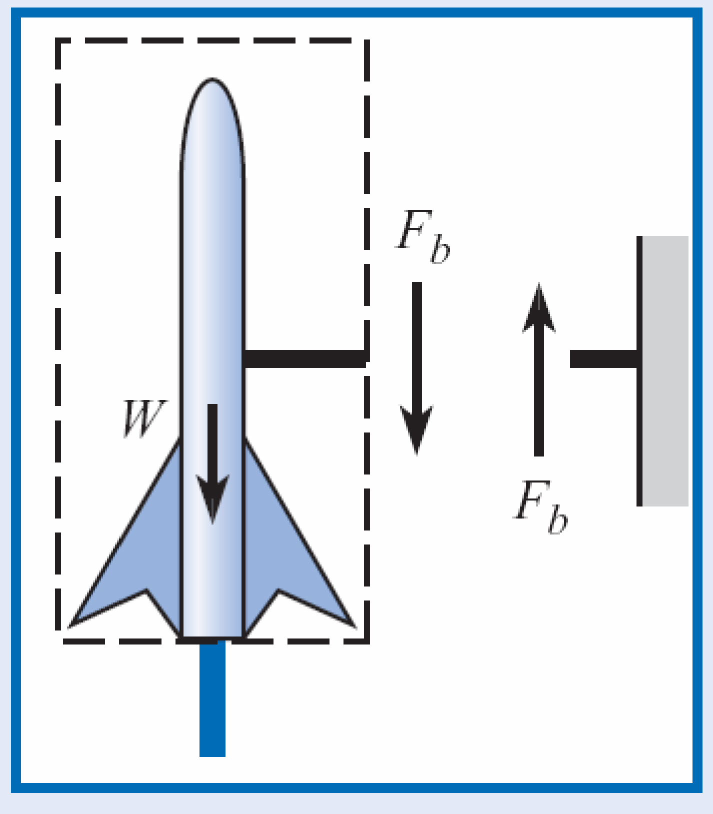
The momentum accumulation

$$\frac{d}{dt}\int_{cv} v_z \rho dQ = 0$$



Z-direction:

$$\sum F_Z = \frac{d}{dt}\int_{cv} v_Z \rho dQ + \sum_{CS}(\dot{m}v)_{outZ} - \sum_{CS}(\dot{m}v)_{inZ}$$



$$\sum F_Z = \sum_{CS}(\dot{m}v)_{outZ} - \sum_{CS}(\dot{m}v)_{inZ}$$

$$= \sum_{CS}(\dot{m}v)_{out} = -\dot{m}v$$

$$-(W + F_b) = -\dot{m}v$$

$$F_b = \dot{m}v - W = (\rho A v)v - mg$$

$$W = mg = (0.04\ \text{kg})(9.81\ \text{m/s}^2) = 0.392\ \text{N}$$

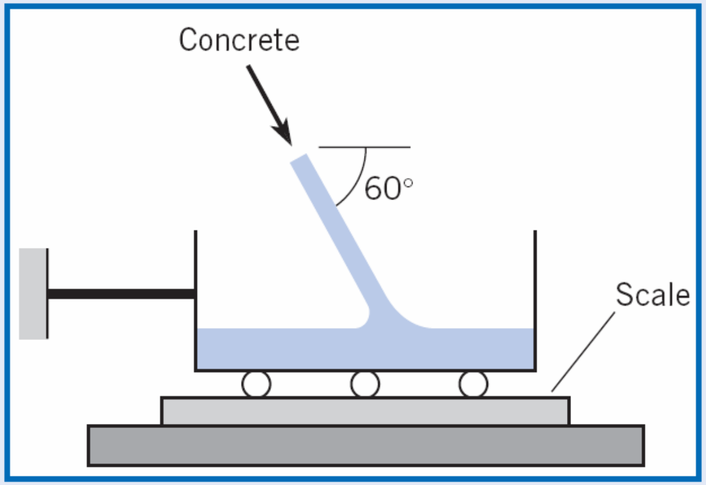
$$\dot{m}v = (\rho A v)v = (0.5\ \text{kg/m}^3)(\pi \times 0.01^2/4\ \text{m}^2)(450^2\ \text{m}^2/\text{s}^2) = 7.95\ \text{N}$$

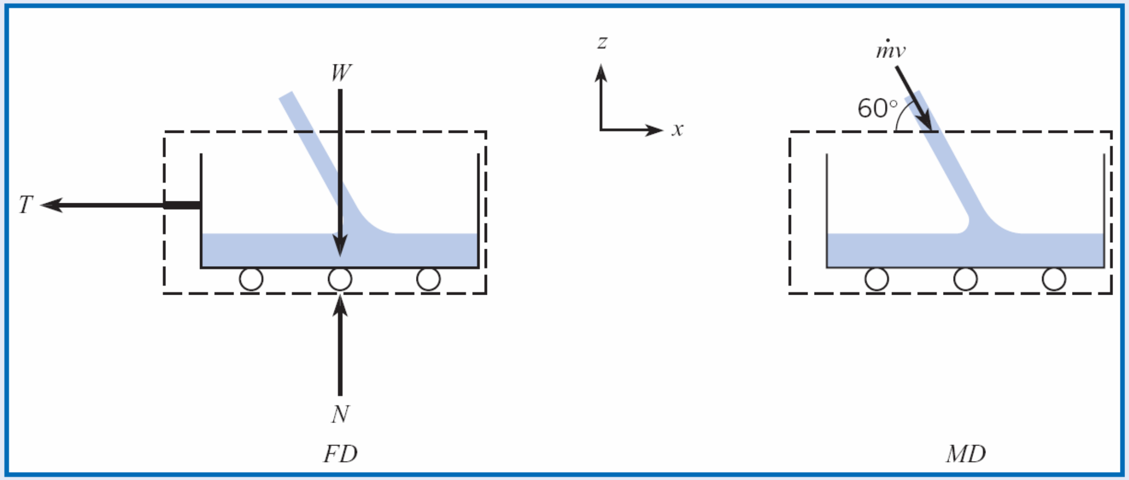
$$F_b = 7.56\,N$$

the direction of F_b (on the beam) is upward.

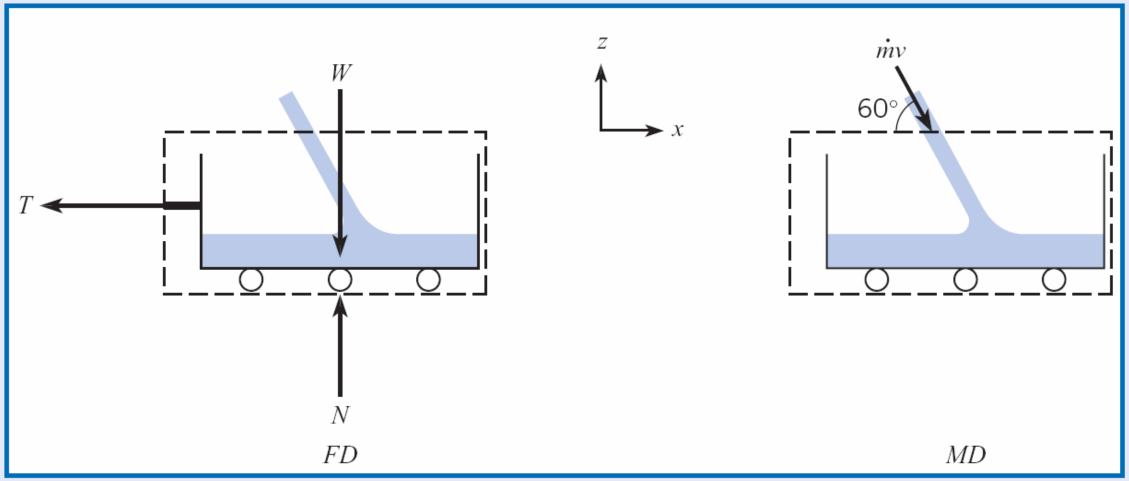
Example (6.2)

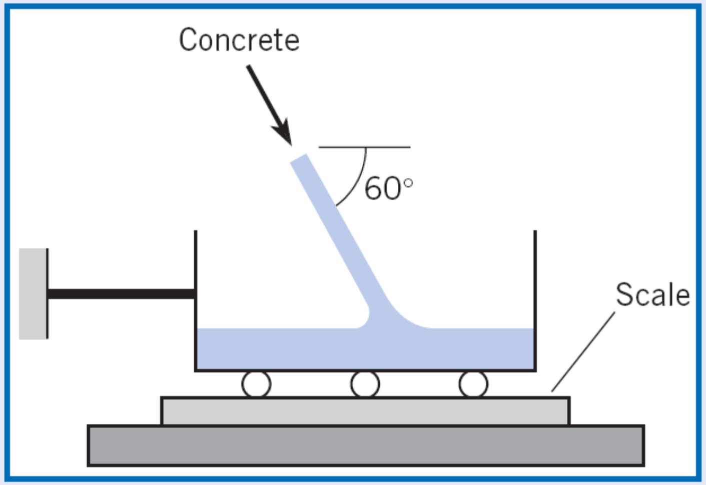
As shown in the sketch, concrete is flowing into a cart sitting on a scale. The stream of concrete has a density of $\rho = 150\ \text{lbm/ft}^3$, an area of $A = 1\ \text{ft}^2$, and a speed of $v = 10\ \text{ft/s}$. At the instant shown, the weight of the cart plus the concrete is 800 lbf. Determine the tension in the cable and the weight recorded by the scale. Assume steady flow.





Find the tension in the cable (T) and the weight recorded by the scale?





Momentum Diagram (MD) **Force Diagram (FD)**

From the diagram above, the problem involves two directions, (x, y)

, therefore, we use the momentum equations in the (x, y)

$$\sum F_X = \frac{d}{dt}\int_{cv} v_X \rho dQ + \sum_{CS} (\dot{m}v)_{outX} - \sum_{CS} (\dot{m}v)_{inX}$$

$$\sum F_Y = \frac{d}{dt}\int_{cv} v_Y \rho dQ + \sum_{CS} (\dot{m}v)_{outY} - \sum_{CS} (\dot{m}v)_{inY}$$

From the force diagram, $\sum F_X = -T$ $\sum F_Y = N - W$

Where N is the weight recorded by the scale

Since the flow is steady and the cart is stationary,

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_Z \rho dQ = 0$

$$\sum F_X = \sum_{CS}(\dot{m}v)_{outX} - \sum_{CS}(\dot{m}v)_{inX} = 0 - \sum_{CS}(\dot{m}v)_{iX} = -(\dot{m}v\cos 60)$$

$$\sum F_Y = \sum_{CS}(\dot{m}v)_{outY} - \sum_{CS}(\dot{m}v)_{inY} = 0 - \sum_{CS}(\dot{m}v)_{iY} = -(-\dot{m}v\sin 60)$$

$$\sum F_Y = N - W = -(-\dot{m}v\sin 60) = (\rho A v)v\sin 60$$

$$\textit{The tension} = T = (\dot{m}v\cos 60) = (\rho A v)v\cos 60$$

$$T = \dot{m}v\cos 60° = 466\ \text{lbf} \times \cos 60° = 233\ \text{lbf}$$

$$\textit{The weight recorded} = N = (\rho A v)v\sin 60 + W$$

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$$\sum F_X = \sum_{CS} (\dot{m}v)_{outX} - \sum_{CS} (\dot{m}v)_{inX} = 0 - \sum_{CS} (\dot{m}v)_{iX} = -(\dot{m}v \cos 60)$$

$$\sum F_Y = \sum_{CS} (\dot{m}v)_{outY} - \sum_{CS} (\dot{m}v)_{inY} = 0 - \sum_{CS} (\dot{m}v)_{iY} = -(-\dot{m}v \sin 60)$$

$$\sum F_X = -T = -(\dot{m}v \cos 60) = -(\rho A v)v \cos 60$$

$$\sum F_Y = N - W = -(-\dot{m}v \sin 60) = (\rho A v)v \sin 60$$

$$\text{The tension} = T = (\dot{m}v \cos 60) = (\rho A v)v \cos 60$$

$$\text{The weight recorded} = N = (\rho A v)v \sin 60 + W$$

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$$\text{The weight recorded} = N = (\rho A v)v\sin 60 + W$$

The momentum flow rate is

$$\dot{m}v = (\rho A v)v$$

$$= (150\ \text{lbm/ft}^3)(1.0\ \text{slug}/32.2\ \text{lbm})(1\ \text{ft}^2)(10^2\ \text{ft}^2/\text{s}^2) = 466\ \text{lbf}$$

The weight recorded by the scale is

$$N = \dot{m}v\sin 60° + W$$

$$= 466\ \text{lbf} \times \sin 60° + 800\ \text{lbf}$$

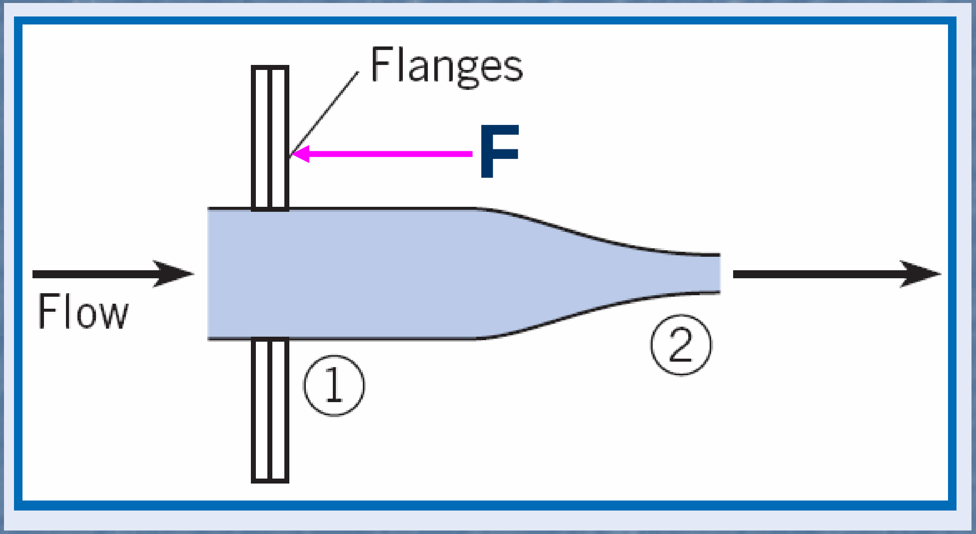
$$= 1200\ \text{lbf}$$

The tension in the cable is

$$T = \dot{m}v\cos 60° = 466\ \text{lbf} \times \cos 60° = 233\ \text{lbf}$$

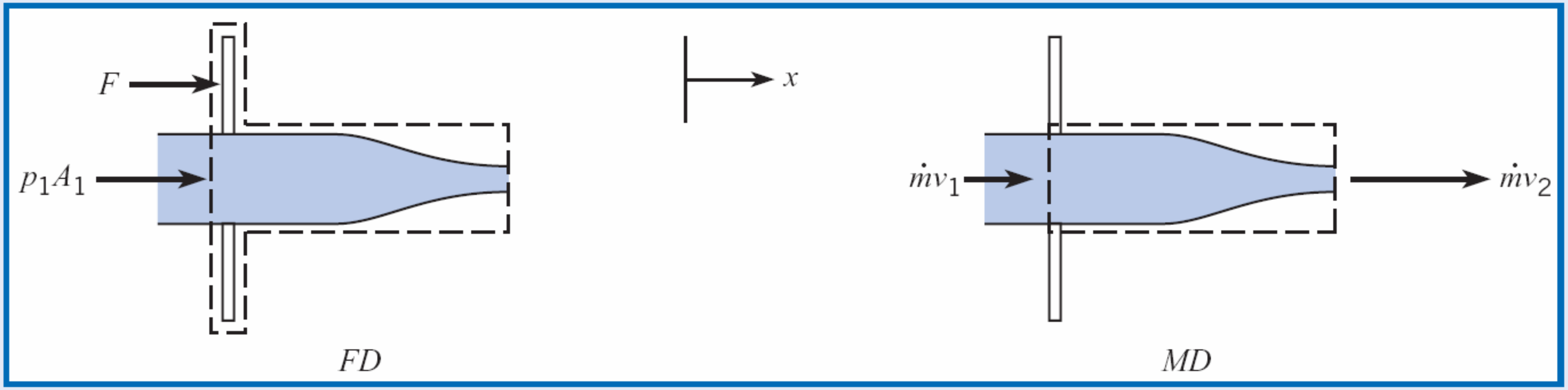
Example 6.3 [Nozzles]

The sketch shows air flowing through a nozzle. The inlet pressure is p_1 = 105 kPa, abs., and the air exhausts into the atmosphere, where the pressure is 101.3 kPa, abs. The nozzle has an inlet diameter of 60 mm and an exit diameter of 10 mm, and the nozzle is connected to the supply pipe by flanges. Find the air speed at the exit of the nozzle and the force required to hold the nozzle stationary. Assume the air has a constant density of 1.22 kg/m³. Neglect the weight of the nozzle.



Find:

The Air Speed (V) at the exit (2)?
The force required to hold the nozzle stationary?



From the diagrams above, the problem involve one direction **(the x-direction)**,

therefore, we use the momentum equations in the **x-direction**

$$\sum F_X = \frac{d}{dt}\int_{cv} v_X \rho dQ + \sum_{CS}(\dot{m}v)_{outX} - \sum_{CS}(\dot{m}v)_{inX}$$

Since the flow is steady and the nozzle is stationary,

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_Z \rho dQ = 0$

From the momentum diagram, $\sum F_X = \sum_{CS}(\dot{m}v)_{outX} - \sum_{CS}(\dot{m}v)_{inX} = \dot{m}v_2 - \dot{m}v_1$

From the force diagram, $\sum F_x = F + p_1 A_1$ $\sum F_x = F + p_1 A_1 = \dot{m}(v_2 - v_1)$

Using Bernoulli's equation and continuity equation to find the velocity at the inlet

$$\text{Bernoulli's Eqn., } p_1 + \gamma z_1 + \frac{\rho v_1^2}{2} = p_2 + \gamma z_2 + \frac{\rho v_2^2}{2}$$

$$Z_1 = Z_2 = 0$$

$$P_2 = P_{atm} = 0$$

$$\text{Continuity Eqn., } A_1 v_1 = A_2 v_2, \quad v_1 = v_2\left(\frac{A_2}{A_1}\right)$$

$$v_2 = \sqrt{\frac{2(p_1 - 0)}{\rho\left(1 - \left(\frac{A_2}{A_1}\right)^2\right)}} = \sqrt{\frac{2(p_1 - 0)}{\rho\left(1 - \left(\frac{d_2}{d_1}\right)^2\right)}}$$

$$v_2 = \sqrt{\frac{2 \times (105 - 101.3) \times 1000 \text{ Pa}}{(1.22 \text{ kg/m}^3)(1 - (10/60)^4)}} = 77.9 \text{ m/s}$$

$$v_1 = v_2 A_2 / A_1 = (77.9 \text{ m/s})(10/60)^2 = 2.16 \text{ m/s}$$

$$\dot{m} = \rho A_2 v_2 = (1.22 \text{ kg/m}^3)(\pi \times 0.01^2/4 \text{ m}^2)(77.9 \text{ m/s})$$

$$= 7.46 \times 10^{-3} \text{ kg/s}$$

END OF LECTURE (2)